AXIAL CURRENT IN QED AND SEMI-NAIVE DIMENSIONAL RENORMALIZATION [1]

M. PERNICI

INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy

M. RACITI

Dipartimento di Fisica dell'Università di Milano, I-20133 Milano, Italy
INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy

ABSTRACT

We renormalize at two loops the axial current and $F\tilde{F}$ in massless QED, using the recently proposed semi-naive dimensional renormalization scheme. We show that the results are in agreement with those in the Breitenlohner-Maison-'t Hooft-Veltman scheme, previously obtained indirectly by making a three-loop computation.

Introduction

Dimensional regularization [1, 2] and minimal subtraction [3] are the most convenient tools for multi-loop computations. However the accuracy of the computations of chiral quantities is much lower than for non-chiral quantities, due to the difficulties encountered in restoring the chiral symmetries in the BMHV scheme [1, 4], the only one known until recently to deal consistently with γ^5 in dimensional regularization.

Few chiral quantities have been computed in the BMHV scheme; among the most accurate computations made up to now there are the renormalization at two loops of the non-singlet axial current [5], of the singlet axial current and of $F\tilde{F}$ [6] in QCD, and the corresponding calculation of the three-loop anomalous dimension of the singlet axial current [6].

The techniques used in these papers to guarantee the validity of the chiral Ward identities in the BMHV scheme are indirect, and not suitable to be generalized to chiral gauge theories. The most impressive trick used is the determination of two-loop finite counterterms in the fermionic sector of the singlet axial current operator by making a three-loop computation in its gluonic sector [6].

Recently it has been shown [7] that there is a consistent extension of the BMHV scheme, called semi-naive dimensional regularization (SNDR), in which after minimal subtraction only few graphs can produce terms breaking the chiral Ward identities. The SNDR scheme has been applied in that paper to the case of the renormalization at two loops of the Yukawa model in presence of external gauge fields, previously studied in the BMHV scheme [8].

As a preliminary investigation in the use of the techniques developed in [8] and [7] for renormalizing gauge theories with chiral interactions, in this letter we apply the SNDR scheme to renormalize at two loops the axial current and $F\tilde{F}$ in QED; to check the chiral Ward identities and to compute the anomalous dimensions we will use the Wilsonian methods introduced in [8].

We find that using the minimal subtraction prescription in SNDR (MS-SNDR) [7] for the axial current the anomalous axial Ward identity is preserved by choosing a non-minimal subtraction for $F\tilde{F}$. Vice-versa, one can satisfy the anomalous axial Ward identity by choosing the minimal subtraction for the operator $F\tilde{F}$ and a non-minimal subtraction for the axial current. With the latter renormalization prescription we find the same three-loop anomalous dimension of the axial current as in the BMHV scheme [6], where minimal subtraction is made on $F\tilde{F}$. Notice that in the BMHV it is not possible to choose minimal subtraction on the axial current, due to the presence

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of the same kind of spurious anomalies appearing in the non-singlet axial current.

In the first section we review the Adler-Bardeen theorem [9], following to a large extent the regularization-independent derivation in [10].

In the second section we review the SNDR scheme and we perform the two-loop renormalization of the axial current and of $F\tilde{F}$ in QED.

In the third section we discuss our methods of computation, we compare our results with those in [6] and we discuss in this context the relation between SNDR and BMHV.

1   Review of the Adler-Bardeen theorem

The classical action of QED with N_f massless fermions in presence of sources for J_μ^5 and K_μ is

$$S^{(0)} = \int \bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2\alpha}(\partial_\mu V_\mu)^2 + ieV_\mu\bar{\psi}\gamma_\mu\psi + A_\mu J_\mu^5 + \chi_\mu K_\mu \quad (1)$$

where we define

$$J_\mu^5 \equiv i\bar{\psi}\gamma_\mu\gamma^5\psi \quad ; \quad K_\mu \equiv 4i\epsilon_{\mu\nu\rho\sigma}V_\nu\partial_\rho V_\sigma \quad (2)$$

We use Euclidean space conventions.

On the functional generator $\Gamma = \Gamma[V, \psi, \bar{\psi}, A, \chi]$ of $1PI$ vertex functions the renormalization group equation reads

$$\mathcal{D}\Gamma = 0 \quad (3)$$
$$\mathcal{D} \equiv \mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial e} + \delta\alpha\frac{\partial}{\partial\alpha} - \sum_i \gamma_i N_i - \int(\gamma_{\chi A}\chi_\mu\frac{\delta}{\delta A_\mu} + \gamma_{A\chi}A_\mu\frac{\delta}{\delta\chi_\mu})$$

where the index i runs over $V_\mu, \psi, \bar{\psi}, A_\mu, \chi$; γ_i and N_i are the corresponding anomalous dimensions and number operators; $\gamma_{A\chi}$ and $\gamma_{\chi A}$ are the mixing anomalous dimensions for the operators (2).

The vectorial Ward identity reads

$$G_v\Gamma = -\frac{1}{\alpha}\partial^2\partial_\mu V_\mu + 4i\epsilon_{\mu\nu\rho\sigma}\partial_\mu\chi_\nu\partial_\rho V_\sigma$$
$$G_v \equiv \partial_\mu\frac{\delta}{\delta V_\mu} + ie\psi\frac{\delta}{\delta\psi} - ie\bar{\psi}\frac{\delta}{\delta\bar{\psi}} \quad (4)$$

Due to the linearity of the breaking terms, they do not need to be renormalized. The proof of this fact in the case of the gauge-fixing term [11] can be straightforwardly extended to the χ term.

The axial Ward identity is

$$G_a\Gamma = \rho\partial_\mu\frac{\delta\Gamma}{\delta\chi_\mu}$$

$$G_a \equiv \partial_\mu\frac{\delta}{\delta A_\mu} + i\gamma^5\psi\frac{\delta}{\delta\psi} + i\bar{\psi}\gamma^5\frac{\delta}{\delta\bar{\psi}} \tag{5}$$

The system of constraints (3,4,5) satisfies the following consistency conditions:

$$[\mathcal{D}, G_v] = i\beta(\psi\frac{\delta}{\delta\psi} - \bar{\psi}\frac{\delta}{\delta\bar{\psi}}) + \gamma_V\partial_\mu\frac{\delta}{\delta V_\mu} \tag{6}$$

$$[\mathcal{D}, G_a - \rho\partial_\mu\frac{\delta}{\delta\chi_\mu}] = (\gamma_A - \rho\gamma_{\chi A})\partial_\mu\frac{\delta}{\delta A_\mu} + (\gamma_{A\chi} - \rho\gamma_\chi - \beta\frac{\partial\rho}{\partial e})\partial_\mu\frac{\delta}{\delta\chi_\mu}$$

which, together with eq.(4) imply respectively

$$\beta = e\gamma_V \;\; ; \;\; \delta = \gamma_\chi = -2\gamma_V \;\; ; \;\; \gamma_{A\chi} = 0 \tag{7}$$

$$\gamma_A = \rho\gamma_{\chi A} \;\; ; \;\; \gamma_\chi = -\beta\frac{\partial ln\rho}{\partial e} \tag{8}$$

from which it follows that, since $\beta \neq 0$,

$$\rho = ca \tag{9}$$

where $a \equiv \frac{e^2}{16\pi^2}$ and c is a constant in e which is fixed by the one-loop anomaly computation to be

$$c = -N_f \tag{10}$$

This proof of the Adler-Bardeen theorem [9], which is close to the one in [10], relies on the validity of (4); this relation can be imposed independently of the regularization scheme. In a regularization which respects the vectorial Ward identities, like Pauli-Villars or the BMHV and the SNDR dimensional regularization schemes, this relation implies that the term $\chi_\mu K_\mu$ of the bare action does not need to be renormalized.

The first relation in (8) gives γ_A at l loops in terms of $\gamma_{\chi A}$ at $(l-1)$-loops.

Allowing for finite renormalization f_α and f_χ respectively for the gauge-fixing and $\chi_\mu K_\mu$ terms, the relations (7) are modified in a trivial way

$$\delta = -2\gamma_V + \beta\frac{\partial ln f_\alpha}{\partial e} \;\; ; \;\; \gamma_\chi = -2\gamma_V + \beta\frac{\partial ln f_\chi}{\partial e} \tag{11}$$

Using (8) one gets

$$\rho = \frac{ca}{f_\chi} \tag{12}$$

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2   Axial current in SNDR

Let us review the BMHV and the SNDR dimensional regularization schemes.

In the BMHV scheme [4] one considers the Lorentz covariants $\delta_{\mu\nu}$, γ_μ, p_μ, etc. as formal objects, satisfying the usual tensorial rules. $\delta_{\mu\nu}$ is the Kronecker delta in $d = 4 - \epsilon$ dimensions; a formal rule for summed indices is given:

$$\delta_{\mu\nu}\delta_{\nu\rho} = \delta_{\mu\rho} \;\; ; \;\; \delta_{\mu\nu}p_\nu = p_\mu \;\; ; \;\; \delta_{\mu\mu} = d \tag{13}$$

The gamma 'matrices' γ_μ satisfy the relation

$$\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}I \tag{14}$$

where I is the identity. The trace is cyclic and satisfies

$$tr\ I = 4 \tag{15}$$

In the BMHV scheme additional '$(d-4)$-dimensional' or 'evanescent' tensors $\hat{\delta}_{\mu\nu}, \hat{p}_\mu$ and $\hat{\gamma}_\mu$ are introduced; the Kronecker delta in the $(d-4)$-dimensional space is $\hat{\delta}_{\mu\nu}$, satisfying

$$\hat{\delta}_{\mu\nu}\delta_{\nu\rho} = \hat{\delta}_{\mu\nu}\hat{\delta}_{\nu\rho} = \hat{\delta}_{\mu\rho} \;\; ; \;\; \hat{\delta}_{\mu\mu} = -\epsilon$$
$$\hat{p}_\mu \equiv \hat{\delta}_{\mu\nu}p_\nu \;\; ; \;\; \hat{\gamma}_\mu \equiv \hat{\delta}_{\mu\nu}\gamma_\nu \tag{16}$$

The Kronecker delta in four dimensions in $\bar{\delta}_{\mu\nu}$, satisfying

$$\bar{\delta}_{\mu\nu} \equiv \delta_{\mu\nu} - \hat{\delta}_{\mu\nu} \;\; ; \;\; \bar{p}_\mu \equiv \bar{\delta}_{\mu\nu}p_\nu \;\; ; \;\; \bar{\gamma}_\mu \equiv \bar{\delta}_{\mu\nu}\gamma_\nu \tag{17}$$

The Levi-Civita antisymmetric tensor has no evanescent component:

$$\hat{\delta}_{\mu\nu}\epsilon_{\nu\rho\sigma\tau} = 0 \tag{18}$$

γ^5 is defined by

$$\gamma^5 = \frac{1}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma \tag{19}$$

which implies

$$\{\gamma^5, \gamma_\mu\} = 2\hat{\gamma}_\mu\gamma^5 \;\; ; \;\; (\gamma^5)^2 = I \tag{20}$$

The fact that γ^5 is not-anticommuting with γ_μ leads to violations of the (non-anomalous) axial Ward identities. On the other hand the possibility of

having anomalous axial currents must be contemplated, so that it is necessary to define γ^5 so that it is not anticommuting. However (20) introduces many breaking terms which are not related to anomalies; these terms are sometimes called spurious anomalies [12].

The SNDR scheme [7] is an extension of this scheme. Add to the BMHV Dirac algebra the objects η and η_1 satisfying the following defining relations:

$$\{\eta, \gamma_\mu\} = \{\eta_1, \gamma_\mu\} = 2\hat{\gamma}_\mu \eta_1$$
$$\eta^2 = I \;\; ; \;\; \eta\eta_1 = \eta_1\eta = \eta_1^2 \;\; ; \;\; \eta_1^3 = \eta_1$$
$$tr \; \eta\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = tr \; \eta_1\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = 4 \; \epsilon_{\mu\nu\rho\sigma} \qquad (21)$$
$$tr \; \eta_1 = tr \; \eta_1^2 = 0 \;\; ; \;\; tr \; \eta_1^2\gamma_{\mu_1}...\gamma_{\mu_r} = 0$$
$$tr \; \eta_1\gamma^5\gamma_{\mu_1}...\gamma_{\mu_r} = tr \; \eta\gamma^5\gamma_{\mu_1}...\gamma_{\mu_r} = tr \; \gamma_{\mu_1}...\gamma_{\mu_r}$$

The trace is cyclic on this enlarged algebra; η_1^2 is a projector.

The idea of this regularization is that using η instead of γ^5 in the tree-level chiral vertices, the number of spurious anomalies is greatly reduced. Let us discuss separately the cases of open and closed fermionic lines belonging to a $1PI$ Feynman graph.

   i) **Open fermionic lines**

In an open fermionic line η can be anticommuted naively modulo terms belonging to the η_1^2 subspace, i.e. monomials containing η_1; algebraic manipulations on a monomial containing η_1 give again terms in the η_1^2 subspace, which cannot be confused with those belonging to the orthogonal subspace. The minimal subtraction on a $1PI$ Feynman graph consists in subtracting all the poles in ϵ and all the finite terms containing η_1. In [7] it is explained why the subtraction of these finite terms is necessary and why it can be considered to be minimal. The idea is that in the process of removing the regulator η is homomorphically mapped in γ^5; since $tr \; (\gamma^5 - \eta)\eta_1 = 4$, there is no way to extend this trace-preserving homomorphism to η_1. Therefore the η_1 terms must be subtracted before applying this homomorphism. Notice that the necessity of subtracting the terms in the η_1^2 subspace has nothing to do a priori with the requirement of satisfying the chiral Ward identities, but it has to do simply with the inner consistency of SNDR.

In the BMHV scheme γ^5 can be anticommuted naively modulo terms containing again γ^5; the latter terms can generate spurious anomalies, which cannot be distinguished from the non-anomalous contributions, so that their subtraction is non-trivial, unlike in SNDR.

   ii) **Closed fermionic lines**

If in a trace there is no η_1 and an even number of η, these η can be anticommuted naively. If in a fermionic trace there is at least one η_1, all the η's in that trace can be replaced with η_1; if at this point there is an
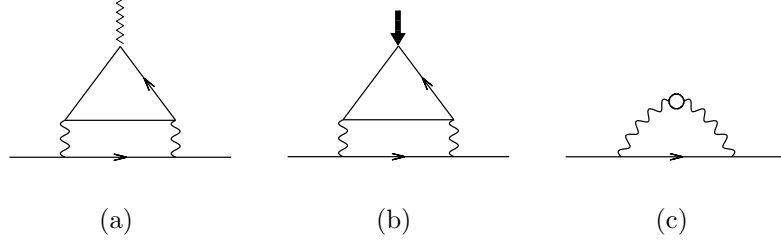
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Figure 1: (a) $\Gamma_{\bar{\psi}\psi A\mu}$; (b) corresponding $\mathcal{T}_a$ contribution; (c) $\Gamma_{\bar{\psi}\psi\chi\mu}$. These are the only graphs for which the axial Ward identity is not trivially satisfied at two loops in SNDR.

even number of η_1's, the trace vanishes; these properties reduce greatly the possibility of occurrence of spurious anomalies.

If in a fermionic trace there is an odd number of η and η_1, they can be replaced with γ^5. The true anomalies originate from these traces.

The bare action in SNDR corresponding to the classical action (1) is

$$S = \int Z_\psi \bar{\psi}\gamma_\mu[\partial_\mu + ieV_\mu + iA_\mu\eta]\psi + \frac{1}{4}Z_V F_{\mu\nu}^2 + \frac{1}{2\alpha}(\partial_\mu V_\mu)^2 +$$
$$iA_\mu\bar{\psi}\gamma_\mu(Z_{A2}\eta_1 + Z_{A3}\gamma^5)\psi + \chi_\mu(K_\mu + Z_{\chi A}i\bar{\psi}\gamma_\mu\gamma^5\psi) \qquad (22)$$

where the sources satisfy $\hat{A}_\mu = 0 = \hat{\chi}_\mu$. The bare action in MS-SNDR is vector gauge invariant apart from tree-level terms; in this scheme the vectorial Ward identities are automatically satisfied. We have computed the two-loop renormalization constants such that the axial Ward identity (5) is satisfied. The only axial Ward identity which is not trivially satisfied is the one involving Figure 1, leading to the non-minimal renormalization

$$Z_{A3}^{non-MS} - \rho Z_{\chi A}^{non-MS} = 3N_f a^2 \qquad (23)$$

All the other renormalization constants can be chosen minimal in SNDR. At

the first two loops the renormalization constants are, for $\alpha = 1$,

$$Z_V = Z_V^{MS} = 1 - \frac{8}{3\epsilon}N_f a - \frac{4}{\epsilon}N_f a^2$$

$$Z_\psi = Z_\psi^{MS} = 1 - \frac{2a}{\epsilon} + (\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2N_f}{\epsilon})a^2$$

$$Z_{A2} = Z_{A2}^{MS} = -4a + (-\frac{16}{3\epsilon} + \frac{4}{9})N_f a^2 + (\frac{8}{\epsilon} + 22)a^2 \tag{24}$$

$$Z_{A3} = \frac{12}{\epsilon}N_f a^2 + Z_{A3}^{non-MS}$$

$$Z_{\chi A} = -\frac{24a}{\epsilon} + (-\frac{64}{\epsilon^2} + \frac{16}{3\epsilon})N_f a^2 + (\frac{48}{\epsilon^2} + \frac{84}{\epsilon})a^2 + Z_{\chi A}^{non-MS}$$

where Z_{A3}^{non-MS} is of order a^2, whereas $Z_{\chi A}^{non-MS}$ is considered at order a only, since its a^2 term is related by the axial Ward identity to the three-loop a^3 term of Z_{A3}^{non-MS}.

The corresponding anomalous dimensions are given by

$$\gamma_V = \frac{4}{3}N_f a + 4N_f a^2$$

$$\gamma_\psi = a - (2N_f + \frac{3}{2})a^2 \tag{25}$$

$$\gamma_{\chi A} = 24a - 72a^2 - \frac{32}{3}N_f a^2 + \frac{16}{3}N_f a Z_{\chi A}^{non-MS}$$

the remaining anomalous dimensions being fixed by eqs.(7, 8). In particular one determines in this way indirectly the three-loop anomalous dimension of the axial current

$$\gamma_A = -24N_f a^2 + 72N_f a^3 + \frac{32}{3}N_f^2 a^3 - \frac{16}{3}N_f^2 a^2 Z_{\chi A}^{non-MS} \tag{26}$$

3 Discussion of the results

To obtain the results in the previous section we have used the Wilsonian method devised in [8] for computing in a systematic way the finite counterterms needed to restore chiral Ward identities in dimensional regularization schemes.

In this approach the renormalization of the theory is obtained imposing renormalization conditions on a Wilsonian functional Γ^Λ, which is perturbatively defined with the same Feynman rules for the vertices as in the usual $1PI$ functional generator Γ, but with the usual propagators D replaced by 'hard' propagators

$$D^H = (1 - K^\Lambda)D \quad ; \quad K^\Lambda(p) = (\frac{\Lambda^2}{p^2 + \Lambda^2})^2 \tag{27}$$

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Let us denote by S_W the local functional whose tree-level part is equal to the classical action (1), and which for $l \geq 1$ is equal to the marginal part of the Wilsonian functional Γ^Λ; one has

$$S_W = \int a_\psi \bar\psi \gamma_\mu \partial_\mu \psi + \frac{1}{2} a_{V1}(\partial_\mu V_\nu)^2 + \frac{1}{2} a_{V2}(\partial_\mu V_\mu)^2 + \frac{1}{4} a_{V4} V_\mu^2 V_\nu^2 +$$
$$i a_{\psi V} V_\mu \bar\psi \gamma_\mu \psi + A_\mu (a_{A\chi} K_\mu + a_A J_\mu^5) + \chi_\mu (a_\chi K_\mu + a_{\chi A} J_\mu^5) \qquad (28)$$

The renormalization group equation and the Ward identities assume now the 'effective' form

$$\mathcal{D} S_W = \mathcal{T}_\gamma \qquad (29)$$

$$\int \epsilon_v G_v S_W = \int \epsilon_v (-\frac{1}{\alpha} \partial^2 \partial_\mu V_\mu + 4 i \epsilon_{\mu\nu\rho\sigma} \partial_\mu \chi_\nu \partial_\rho V_\sigma) + \mathcal{T}_v \qquad (30)$$

$$\int \epsilon_a G_a S_W = \int \epsilon_a \rho \partial_\mu \frac{\delta S_W}{\delta \chi_\mu} + \mathcal{T}_a \qquad (31)$$

where the $\mathcal{T}$ terms are contributions to the number operators and to the contact term operators at the Wilsonian scale. The Green functions of the $\mathcal{T}$ terms can be computed using Feynman rules as described in [8].

We use the hard propagator $D_{\mu\nu}^H = D^H \delta_{\mu\nu}$ for the photon in the Feynman gauge.

To compute the renormalization group functions we use (29). The derivative with respect to the gauge-fixing parameter in (29) in $\alpha = 1$ is treated as the insertion of the operator $\int \frac{1}{2}(\partial_\mu V_\mu)^2$.

As discussed before, the vectorial Ward identities are trivially satisfied in MS-SNDR, so that we have to discuss only the axial Ward identities (31).

One has

$$\mathcal{T}_a = \int \partial_\mu \epsilon_a (b_\psi J_\mu^5 + b_\chi K_\mu) \qquad (32)$$

The axial Ward identity gives the following relations

$$a_A - a_\psi + b_\psi - \rho a_{\chi A} = 0 \qquad (33)$$
$$a_{A\chi} + b_\chi - \rho a_\chi = 0 \qquad (34)$$

The marginal part of S_W exists for $\Lambda > 0$; we will compute its coefficients at $\Lambda = \mu$, the dimensional regularization scale.

At zero and one loops one has, in the MS-SNDR scheme,

$$a_\psi^{MS} = 1 + \frac{a}{6} \quad ; \quad a_A^{MS} = 1 - \frac{43a}{60}$$
$$a_{\chi A}^{MS} = \frac{77a}{5} \quad ; \quad a_{A\chi}^{MS} = -\frac{2N_f a}{3} \quad ; \quad a_\chi = 1 \qquad (35)$$
$$b_\psi = \frac{53a}{60} \quad ; \quad b_\chi = -\frac{N_f a}{3}$$

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The axial Ward identities (33,34) are manifestly satisfied in the SNDR scheme as long as γ^5 or the Levi-Civita tensor does not appear in the Feynman rules. This is the case at one loop, with the exception of the $\Gamma_{\bar\psi\psi\chi\mu}$ vertex, which is related by the axial Ward identity to the two-loop insertions of $\Gamma_{\bar\psi\psi A\mu}$, so that we will discuss it later.

To illustrate this renormalization procedure, consider for instance the relevant part of the unrenormalized Wilsonian axial vertex $\Gamma^{unren\Lambda}_{\bar\psi\psi A\mu}$ at one loop:

$$ia[(\frac{2}{\epsilon} - \frac{43}{60})\bar\gamma_\mu\eta + 4\bar\gamma_\mu\eta_1] \tag{36}$$

The pole term and the η_1 terms are minimally subtracted, giving the renormalization constants Z_ψ and Z_{A2} at one loop (24); the remaining finite part, evaluated in four dimensions (i.e. $\eta \to \gamma^5$), gives the contribution to S_W in (28,35).

At two loops the MS-SNDR scheme satisfies manifestly the axial Ward identities, apart from the contribution of Fig.1. The two-loop Wilsonian Green function $\Gamma^{unren\Lambda}_{\bar\psi\psi A\mu}$, unrenormalized at two loops but renormalized at one loop, gives the following relevant contributions

$$ia^2\bar\gamma_\mu\{-(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{806251}{204120} + \frac{12937v}{4374})\eta - (\frac{8}{\epsilon} + 22)\eta_1 -$$
$$N_f(\frac{2}{\epsilon} - \frac{1363}{5670} + \frac{1136v}{243})\eta + N_f(\frac{16}{3\epsilon} - \frac{4}{9})\eta_1 - N_f(\frac{12}{\epsilon} + \frac{2449}{270} + \frac{1072v}{81})\gamma^5\}$$

where the $\bar\gamma_\mu\gamma^5$ contribution, which comes from the graph in Figure 1(a), is the only one for which minimal subtraction is not automatically sufficient to preserve the axial Ward identity. The pole terms and the η_1 terms are minimally subtracted, giving the renormalization constants Z_ψ, Z_{A2} and Z^{MS}_{A3} at two loop (24).

Let us consider for instance the contributions to the axial Ward identity due to the graphs in Figure 2 and the corresponding counterterms:

$$a^{MS}_A(fig.2) = (\frac{1363}{5670} - \frac{1136v}{243})N_f a^2$$
$$b^{MS}_\psi(fig.2) = -(\frac{28393}{17010} + \frac{2272v}{729})N_f a^2$$
$$a^{MS}_\psi(fig.2) = -(\frac{1736}{1215} + \frac{5680v}{729})N_f a^2$$

There is no K_μ insertion corresponding to the graphs in Figure 2, so that $a^{MS}_{\chi A}(fig.2) = 0$ and the axial Ward identity (33) is satisfied as expected.

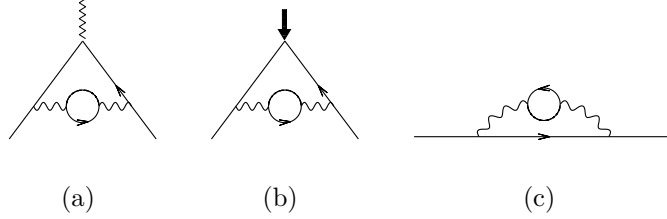<div align="center">(a)   (b)   (c)</div>

Figure 2: Example of graphs for which the axial Ward identity is trivially satisfied at two loops in SNDR.

Let us consider finally the only axial Ward identity which is not automatically satisfied in this scheme. For the graph in Fig. 1 one has

$$a_A^{MS}(fig.1) = -(\frac{2449}{270} + \frac{1072v}{81})N_f a^2$$
$$b_\psi^{MS}(fig.1) = (-\frac{2519}{270} + \frac{1072v}{81})N_f a^2$$
$$a_\psi^{MS}(fig.1) = 0$$

The corresponding K_μ insertion gives $a_{\chi A}^{MS}(fig.1) = \frac{77a}{5}$ (see eq.(35)) so that in the minimal scheme the axial Ward identity (33) is not satisfied

$$(a_A - a_\psi + b_\psi - \rho a_{\chi A})^{MS}(fig.1) = -3N_f a^2 \qquad (37)$$

and the non-minimal counterterms in (23) must be added to restore (33).

To obtain the bare action in the BMHV scheme, one can use the values found for S_W as renormalization conditions at the Wilsonian scale. The result is simply phrased: it is sufficient to replace η and η_1 in (22) with γ^5. In fact, if η or η_1 belongs to an open fermionic line of a $1PI$ graph, anticommuting it through the gamma matrices of the fermionic lines one uses only the relations in the first line of (21), which agree with (20) after replacing η and η_1 with γ^5. If η or η_1 belongs to an open fermionic line of a $1PI$ graph, it gives the same as in the case in which it is replaced by γ^5, due to the last line in (21).

The correspondence between the bare actions in these two schemes is not always so simple; in presence of more than one chiral vertex the BMHV scheme produces many finite counterterms which are absent in the SNDR scheme; the difference is due to the fact that $tr\ \eta_1^2 = 0$ in SNDR (see (21)) whereas $tr\ (\gamma^5)^2 = 4$ in BMHV; see for instance the Yukawa model in the BMHV scheme [8] and in the SNDR scheme [7].

Our results agree with those in [6], after the changes due to conventions, provided one makes the minimal choice for $Z_{\chi A}$. Similarly for the anomalous

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dimensions. In the BMHV scheme it is possible to make this minimal choice, whereas one cannot make minimal subtraction of the axial current, since the one-loop finite counterterm $\int 4aA_\mu J_\mu^5$ in the bare action necessary to satisfy the axial Ward identity cannot be replaced by the term $\int \frac{4}{N_f}\chi_\mu J_\mu^5$ without violating the classical limit of the operator K_μ.

On the other hand in the SNDR scheme it is possible to make the minimal subtraction either on K_μ or on J_μ^5, to all orders in perturbation theory.

Let us review the tricks used in [6] to perform this computation. To compute the one-loop finite counterterm for the axial current, comparison between the axial and the vector vertices is made, as suggested in [12]. To compute the two-loop finite counterterm the same trick cannot be used in the case of the singlet axial current, due to fact that the graph in Figure 1 has no counterpart in the vector vertex. To fix this finite counterterm, instead of checking directly the axial Ward identity on the axial vertex at two loops, the three-loop computation of $< \partial_\mu J_\mu^5 V_\nu V_\rho >$ has been made in [6], obtaining these two-loop finite terms by consistency with the Adler-Bardeen theorem.

4   Conclusion

The SNDR scheme is a consistent extension of the BMHV dimensional regularization and renormalization scheme, which has been introduced to reduce the number of spurious anomalies present in the latter scheme.

As a preliminary investigation in gauge theories with chiral couplings, in this letter we have applied the SNDR scheme to the renormalization of the axial current in QED. In this case there is only one chiral vertex, so that there are few spurious anomalies, which have been determined in the BMHV scheme in [6], together with the three-loop anomalous dimension of the singlet axial current in QCD. We find agreement with these results in the QED case. The correspondence between the SNDR and the BMHV scheme is in this case so easy that it can be made even at the bare action level. As expected, we found that it is easier to satisfy the axial Ward identity in the SNDR scheme than in the BMHV scheme. These computational advantages are expected to be much greater in the case of chiral gauge theories, which have been renormalized systematically in the BMHV scheme only at one loop [13].

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